

# Vacua Landscape Attractor

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## Abstract

The recent progress in the understanding of the landscape of string theory vacua hints that the hierarchy problem might be the problem of a super-selection rule. The attractor mechanism gives a possibility to explain the choice of a vacuum. We consider a toy model of self-interacting membranes and show that for a very generic interaction there are attractor solutions.

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# 1 Introduction

The hierarchy problem manifests itself in the enormous difference between the standard model, gravity and in a wider sense the dark energy scales. It is assumed often that in the case of the standard model its solution requires some UV regulating physics. However, as it was suggested in [1]-[2], the hierarchy problem can be addressed as the problem of a vacuum super-selection rule. The recent progress in the understanding of the string theory vacua landscape (see for example [3]) gives a hint on the different possibilities of vacua density distributions. This motivates the studying of an alternative mechanism as a possible solution to the hierarchy problem. The idea proposed in [2], based on an earlier work on cosmic attractors [1], is to put to work the multiplicity of vacua. The hierarchy problem is promoted into a problem of the super-selection rule among the infinite number of vacua, that are finely scanned by the Higgs mass. In this framework, the Higgs mass is promoted into a dynamical variable. An infinite number of vacua cluster around a certain point making it an attractor. On the resulting landscape in all but a measure zero set of vacua the Higgs mass has a common, hierarchically small value due to the attractor.

In this paper we will analyze a model which can be viewed as an effective theory obtained after integrating out the Higgs field in the model [2]. As a result we get a setup in which branes adjust their charges according to the values of the field they produce. We consider the possibility of having an attractor for such system. Three-forms are sourced out by a number of membranes (two-branes) with charges  $q$  that can be self-adjusted. In particular, we will consider the membrane charges being tuned as an arbitrary function of the field. This problem essentially can be reduced to that of field dependent charges in 1+1 electrodynamics. As we will show, the presence of an attractor is a very generic feature of such models.

## 2 Three-forms and two-branes

Let's review the setup of the work [2]. The spectrum of different string theories contains antisymmetric form fields, which after compactification to four dimensions give rise to three-forms, two-forms and one-forms. In particular, we are interested in three-forms  $C_{\alpha\beta\gamma}$ . The action for a three-form in four

dimensions reads

$$S_C = \int_{3+1} \frac{1}{48} F_{\mu\alpha\beta\gamma} F^{\mu\alpha\beta\gamma}, \quad (1)$$

where the four-form field strength

$$F_{\mu\alpha\beta\gamma} = d_{[\mu} C_{\alpha\beta\gamma]}. \quad (2)$$

This action is gauge invariant, and this guarantees the decoupling of the time component. The gauge transformation is

$$C_{\alpha\beta\gamma} \rightarrow C_{\alpha\beta\gamma} + d_{[\alpha} \Omega_{\beta\gamma]}, \quad (3)$$

where  $\Omega_{\beta\gamma}$  is some two-form depending on the coordinates and the square brackets denote anti-symmetrization. The form  $C$  has no propagating degrees of freedom in four dimensions. The equations of motion stemming from the action (1) are

$$\partial^\mu F_{\mu\nu\alpha\beta} = 0 \quad (4)$$

and have a constant solution

$$F_{\mu\nu\alpha\beta} = F_0 \epsilon_{\mu\nu\alpha\beta}; \quad (5)$$

here  $F_0 = \text{constant}$  and  $\epsilon_{\mu\nu\alpha\beta}$  is the totally antisymmetric tensor. In the absence of interactions with other fields this constant changes the Lagrangian and contributes to the cosmological term. In the presence of interactions  $F_0$  will contribute to those fields masses and to the couplings.

Three-forms couple to two-branes, e.g. membranes. The effective action is given by

$$S = \frac{q}{6} \int_{2+1} d^3\xi C_{\mu\nu\alpha} \left( \frac{\partial Y^\mu}{\partial \xi^a} \frac{\partial Y^\nu}{\partial \xi^b} \frac{\partial Y^\alpha}{\partial \xi^c} \right) \epsilon^{abc} - \int_{3+1} \frac{1}{48} F^2. \quad (6)$$

where  $q$  is the brane charge and  $Y^\mu(\xi)$  describe the brane history as a function of its world volume coordinates  $\xi^a$ ,  $a = 0, 1, 2$ . We can rewrite the interaction term as a four dimensional integral

$$\int d^4x \frac{1}{6} J^{\alpha\beta\gamma} C_{\alpha\beta\gamma} \quad (7)$$

where the brane current

$$J^{\alpha\beta\gamma}(x) = \int d^3\xi \delta^4(x - Y(\xi)) q \left( \frac{\partial Y^\alpha}{\partial \xi^a} \frac{\partial Y^\beta}{\partial \xi^b} \frac{\partial Y^\gamma}{\partial \xi^c} \right) \epsilon^{abc}. \quad (8)$$

As long as  $q$  is constant, the current is conserved. We end up with the equations of motion

$$\partial_\mu F^{\mu\nu\alpha\beta} = -q \int d^3\xi \delta^4(x - Y(\xi)) \left( \frac{\partial Y^\nu}{\partial \xi^a} \frac{\partial Y^\alpha}{\partial \xi^b} \frac{\partial Y^\beta}{\partial \xi^c} \right) \epsilon^{abc}. \quad (9)$$

We consider the simple case of static and flat branes,

$$Y^\mu = \xi^\mu, \quad \mu = 0, 1, 2 \quad (10)$$

$$Y^3 = 0 \quad (11)$$

We take  $x_3 = z$  as the coordinate transversal to the brane. Then the equations of motion reduce to

$$\partial_\mu F^{\mu\nu\alpha\beta} = -q \delta(z) \epsilon^{\nu\alpha\beta z}. \quad (12)$$

The equations of motion show that the brane separates two vacua. In each of them the field strength is constant and the jump between the values of the field in different vacua is given by the brane charge  $q$ . This way, there is a solution with multiplicity of vacua and the vacua in this solution are labeled by an integer  $n$ ,

$$-\frac{1}{24} F_{\alpha\beta\gamma\mu} \epsilon^{\alpha\beta\gamma\mu} = qn + F_0, \quad (13)$$

where  $F_0$  is a constant which in the theory with an attractor mechanism will be fixed.

In the model [2], the lowest order parity and gauge-invariant Lagrangian describing a non-trivial interaction between the Higgs field  $\phi$  and the gauge field  $C$  was suggested in the form

$$L = |\partial_\mu \phi|^2 - \frac{1}{48} F^2 + |\phi|^2 \left( m^2 + \frac{F^2}{48M^2} \right) - \frac{\lambda}{2} |\phi|^4 + \dots \quad (14)$$

where  $\lambda$  is the quartic coupling and  $m, M$  are mass parameters. As a result, the gauge field determines the value of the effective mass and consequently the vacuum expectation value of the Higgs field. The Higgs field in turn readjusts the brane charges and gauge field closing a cycle. This can create an attractor depending on details of the interaction provided there is an additional symmetry forbidding higher loop corrections to the classical attractor.

### 3 Explicit gauge field dependence

The key idea is to consider a charge  $q(F)$  being explicitly field dependent; this corresponds to effective “integrating out” the Higgs mass in the model [2]. To simplify the derivations and make the physical content clearer, we will consider the 1+1 case and will call the field potential  $A$ . Thus we will have electrodynamics with self-adjusting charges. In the absence of the mentioned dependence the current is

$$J^\mu(x) = \int d\xi q \delta^2(x - Y(\xi)) \frac{\partial Y^\mu(\xi)}{\partial \xi}, \quad (15)$$

where the charge  $q$  acts as source for the gauge field. If we consider the charges being field depending sources, the field in turn readjusts the charge. The corresponding current can be written as

$$J^\mu(x) = \int d\xi q(F(Y(\xi))) \delta^2(x - Y(\xi)) \frac{\partial Y^\mu(\xi)}{\partial \xi}. \quad (16)$$

This current is no longer conserved unless we rewrite the interaction so that the field couples only to the transverse part of the current. This can be guaranteed by an interaction term with a projection kernel

$$\Pi_{\mu\nu} = g_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2} \quad (17)$$

so that the Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + A^\mu \Pi_{\mu\nu} J^\nu. \quad (18)$$

The potential  $A_\mu$  couples only to the transverse part of the current  $J^\mu$ . For a single static charge current located at point  $x_1 \equiv z = a$

$$J^0(z) = q(F(z)) \delta(z - a), \quad J^1(z) = 0 \quad (19)$$

The interaction term

$$J^0 \Pi_{0\nu} A^\nu = J^0 \Pi_{00} A^0 = J^0 A_0 - J^0 \frac{\partial_0 \partial_0}{\partial^2} A_0 = J^0 A_0 \quad (20)$$

and the Lagrangian becomes

$$\mathcal{L} = -\frac{1}{2} (\partial_z A_0)^2 + q(\partial_z A_0) \delta(z - a) A_0. \quad (21)$$

The variation of the Lagrangian is

$$\delta\mathcal{L} = (\partial_z^2 A_0 + q(\partial_z A_0)\delta(z-a) - \partial_z(q'(dzA_0)\delta(z-a)A_0))\delta A_0 \quad (22)$$

and gives the equation of motion

$$\begin{aligned} \partial_z^2 A_0 = \delta(z-a) & \left[ \partial_z A_0 q'(\partial_z A_0) - q(\partial_z A_0) + q''(\partial_z A_0)\partial_z^2 A_0 A_0 \right] + \\ & + \partial_z \delta(z-a) q'(\partial_z A_0) A_0. \end{aligned} \quad (23)$$

Outside the brane the equation of motion reduces to

$$\partial_z^2 A_0 = 0. \quad (24)$$

The general solution for the field strength is a constant,

$$F \equiv F_{10} = \partial_z A_0 = \text{const.} \quad (25)$$

Integrating in a small neighborhood near the brane leads to the boundary condition

$$F(a+0) - F(a-0) = \quad (26)$$

$$= [Fq'(F) - q(F) + q''(F)\partial_z F A_0 - \partial_z(q'(\partial_z A_0)A_0)]_{z=a} \quad (27)$$

or equivalently

$$F(a+0) - F(a-0) = -q(F)|_{z=a}. \quad (28)$$

We can implement this boundary condition into the equation of motion as

$$\partial_z F = -q(F(a))\delta(z-a) \quad (29)$$

Let's look at the behavior of the field when we add in succession  $N$  charged branes at points  $z = a_n$  (for convenience we take  $a_n > a_k$  for  $n > k$ ). In this case we get the following equation

$$\partial_z F = - \sum_{k=1}^N q(F(a_k))\delta(z-a_k) \quad (30)$$

where the integer  $k = 1, \dots, N$  labels the branes. The general solution to this equation is

$$F(z) = - \sum_{k=1}^N q(F(a_k))\theta(z-a_k) \quad (31)$$

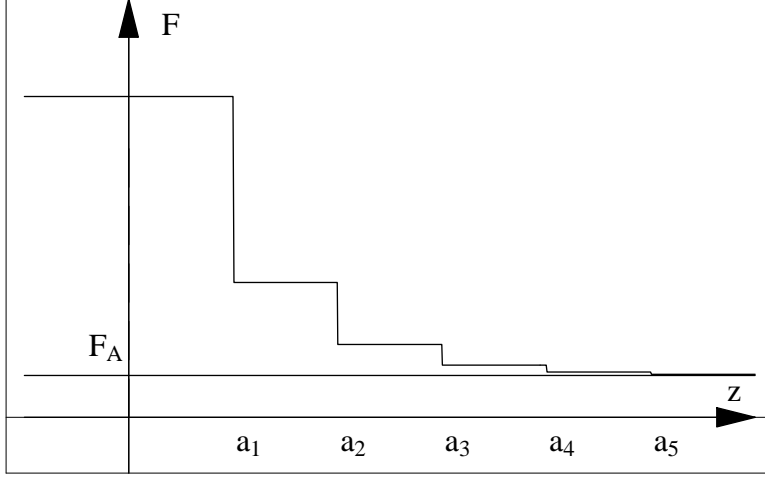


Figure 1: Field configuration with field-dependent charges,  $0 < c < 2$ .

with the boundary condition to be applied. These boundary conditions will fix the constants  $q(F(a_k))$ . The equations of motion lead to the following recursion relation

$$F_k + \frac{1}{2}q(F_k) = F_{k-1} - \frac{1}{2}q(F_{k-1}) \quad (32)$$

for the values of the fields between the branes. As  $N \rightarrow \infty$ , if there is an attractor point, the following limit should exist

$$\lim_{N \rightarrow \infty} F_N \rightarrow F_A. \quad (33)$$

From the recursion one can see that when the limit exists, then  $q(F_A) = 0$ . It means that the attractor point candidate  $F_A$  should be a root of the equation

$$q(F) = 0. \quad (34)$$

In Figures 1 and 2 we show how the addition of branes at points  $a_1, a_2, \dots$  leads to an increasing number of vacua near the attractor point  $F = F_A$ . A sufficient condition for the existence of the above limit, i.e. the attractor point at the root  $F_A$ , is  $q'(F_A) > 0$ .

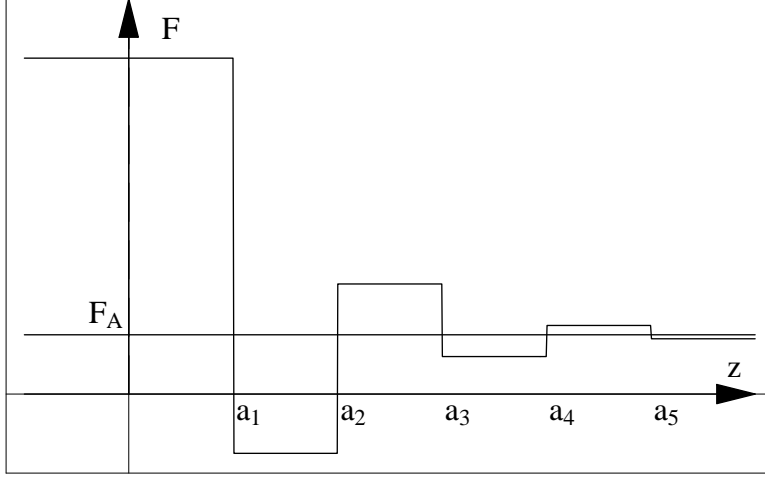


Figure 2: Field configuration with field dependent charges,  $c > 2$ .

## 4 Number Density near the Attractor Point

We will evaluate the vacuum number density near the attractor point. In the case of a charge depending linearly on the field  $F$ ,

$$q(F) = c(F - F_A) \quad (35)$$

with  $c$  some positive constant, the recursion (32) with an initial value  $F = F_1$  has the solution

$$F_k = \left( \frac{2-c}{2+c} \right)^{k-1} (F_1 - F_A) + F_A. \quad (36)$$

We can express the number of vacuum states outside the interval of fields  $F - F_A$

$$k = 1 + \frac{\ln \frac{F-F_A}{F_1-F_A}}{\ln \frac{2-c}{2+c}}. \quad (37)$$

Correspondingly, the number density of the vacuum states in the linear case is given by

$$n \sim -\frac{dk}{dF} = -\frac{1}{\ln \frac{2-c}{2+c}} \frac{1}{|F - F_A|} \quad (38)$$

and is divergent at the attractor point  $F = F_A$ .



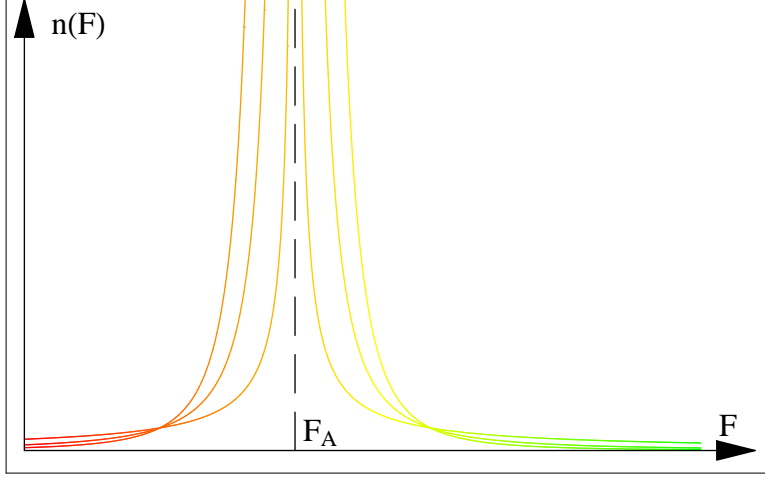


Figure 3: The number of states density around the attractor point.

We would like to calculate the number density for arbitrary self-interaction but we cannot explicitly write the number of states  $k$  in terms of the field range for a generic function  $q(F)$ . Nevertheless we can estimate the derivative via the recursion relation

$$n \sim \frac{\Delta k}{\Delta F} = \frac{1}{F_k - F_{k-1}} \sim \frac{1}{q(F)} \quad (39)$$

This number of states is also divergent at the attractor point. In Figure (3) we have depicted the typical, divergent behavior of the number density of vacua near the attractor point.

Let's find the domain of convergence to the attractor. The answer comes from the sufficient condition of the attractor existence at the point  $F = F_A$

$$q(F_A) = 0, \quad q'(F_A) > 0. \quad (40)$$

It follows from the recursion relation that in the vicinity of the attractor point  $F_A$  to guarantee the convergence we should continuously satisfy

$$q'(F) > 0, \quad (41)$$

$$q'(F) < 4 + q'(F_A). \quad (42)$$

In the linear example

$$q(F) = c(F - F_A) \quad (43)$$

the fields converge everywhere for  $c > 0$ .

For a quadratic dependence

$$q(F) = (F - F_A)(F - F_B) \quad (44)$$

the attractor will be located at  $F_A$  for

$$F_A > F_B. \quad (45)$$

The range of the fields  $F$  which converge to the attractor point is defined by

$$0 < q'(F) < 4 + q'(F_A). \quad (46)$$

This restricts  $F$  to the range

$$(F_A + F_B)/2 < F < 2 + F_A. \quad (47)$$

## 5 Conclusions

We have shown that the self-adjusting charges or membranes have an attractor point. This implies that the number density of vacua within a small range around the attractor point blows up. From the physical point of view, the attractor adjusts to the point with a minimal self-interaction, creating an enormous number of vacua with close values. The sufficient condition for the attractor existence is vanishing of the charge at the attractor point and the charge being an increasing function of the field. Even if the interaction never reaches zero, this point still will be like an attractor. However, the number density of vacua will have a finite maximum sharp pick.

We hope that the suggested attractor model can be implemented for a natural explanation of large hierarchies, like scales of electroweak theory, gravity and dark energy.

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